It seems to us that this weaker conjecture is provable, but we have not proved it. While (6) has not been proven, one can also examine the sequences

$$p, p + 2k$$

collectively, for all k. This has been done by Lavrik [5], and results have been obtained there concerning "almost all" k. If the generalization suggested in (A) is carried out successfully, it seems to us that Lavrik's techniques applied to our (3) should suffice to prove that there are infinitely many Mersenne composites, and probably also stronger results concerning a lower bound on their number. Further, one would then also have an upper bound on the number of Mersenne primes.

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A Counterexample to Euler's Sum of Powers Conjecture

By L. J. Lander and T. R. Parkin

A search was conducted on the CDC 6600 computer for nontrivial solutions in nonnegative integers of the Diophantine equation

$$x_1^5 + x_2^5 + \cdots + x_n^5 = y^5, \qquad n \leqq 6. \tag{1}$$

In general, to decompose t as the sum of n fifth powers assume s is the largest. Then for each s in the range

$$(t/n)^{1/5} \leqq s \leqq t^{1/5},$$

a decomposition is sought in which $t - s^5$ is the sum of $n - 1$ fifth powers each $\leqq s^5$. Applying the algorithm repeatedly a final decomposition is reached of the form

$$u = v^5 + w^5$$

in which $w \leqq v$ and each v in the range $(u/2)^{1/5} \leqq v \leqq u^{1/5}$ is considered. Since $x^5 \equiv x \pmod{30}$ for each integer x, we require $w \equiv u - v \pmod{30}$. A precalculated

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TABLE 1
All Primitive Solutions of $x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + x_6^5 = y^5$, $y \leqq 100$

x_1	x_2	x_3	x_4	x_5	x_6	y
4	5	6	7	9	11	12
5	10	11	16	19	29	30
15	16	17	22	24	28	32
13	18	23	31	36	66	67
7	20	29	31	34	66	67
0	19	43	46	47	67	72
22	35	48	58	61	64	78
0	21	23	37	79	84	94
4	13	19	20	67	96	99
6	17	60	64	73	89	99

TABLE 2
All Primitive Solutions of $x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = y^5$, $y \leqq 250$

x_1	x_2	x_3	x_4	x_5	y
19	43	46	47	67	72
21	23	37	79	84	94
7	43	57	80	100	107
0	27	84	110	133	144

table of fifth powers was employed and table lookup replaced the taking of fifth roots in determining limits on the x_i.

For $n = 6$, there are only ten primitive solutions of (1) in the range $y \leqq 100$ and these are given in Table 1. The least two of these were obtained by A. Martin [1]. Among the new solutions was

$$19^5 + 43^5 + 46^5 + 47^5 + 67^5 = 72^5$$

which is the least solution of (1) with $n = 5$. The search was then specialized to $n = 5$, and the four solutions given in Table 2 are the only primitive solutions over the range $y \leqq 250$. The third case ($y = 107$) is the least solution given by Sastry's identity [2]

$$(75\,v^5 - u^5)^5 + (u^5 + 25\,v^5)^5 + (u^5 - 25\,v^5)^5 + (10\,u^3v^2)^5 + (50\,uv^4)^5 = (u^5 + 75\,v^5)^5$$

for $u = 2$, $v = 1$. The fourth case was the unexpected result.

$$27^5 + 84^5 + 110^5 + 133^5 = 144^5$$

which is a counterexample to Euler's conjecture [3] that at least k positive kth powers are required to sum to a kth power, except for the trivial case of one kth power: $y^k = y^k$. The search was again specialized to $n = 4$ over the range $y \leqq 750$, but no further primitive solutions exist in that range.

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Statistics on Certain Large Primes

By M. F. Jones, M. Lal and W. J. Blundon

Introduction. In any heuristic approach to the distribution of prime numbers, lists of primes are necessary and it is to this end that many such lists have been prepared. For primes up to $37 \cdot 10^6$, Lehmer [1] has made detailed comparisons with some of the conjectures of Hardy and Littlewood [2] and, more recently, primes up to 104395289 have been listed [3] and statistics compiled [4].

The purpose of this investigation has been to determine whether the conjectures concerning the number of primes, twins, triples and quadruples in a given interval x to $x + \Delta x$ continue to hold for somewhat larger x than have previously been considered.

The following ranges were chosen:

$$10^n \to 10^n + 150{,}000, \qquad n = 8(1)15,$$

and the primes were found using the following algorithm.

Let us define a sequence $\{S_k\}$ of k odd numbers where

$$S_k = x + 2(k-1) \leqq p_n^2$$

with $p_i = 2i + 1$, $i = 1, 2, \cdots n$ and $x > p_n$, and a further sequence $\{r_i\}$ such that

$$x \equiv r_i \pmod{p_i}.$$

Then all composite numbers in the sequence $\{S_k\}$ fall in the sequence $\{C_{im}\}$ where

$$C_{im} = x + (a_i \cdot p_i - r_i) + 2mp_i, \qquad m = 0, 1, \cdots$$

with

$$\begin{aligned} a_i &= 0 \quad \text{for } r_i \quad \text{zero} \\ &= 1 \qquad\qquad\ \ \text{even}, \qquad i = 1, 2, \cdots n. \\ &= 2 \qquad\qquad\ \ \text{odd} \end{aligned}$$

The above algorithm was coded for an IBM 1620 with 40K core storage, and by representing two odd numbers by one core address,* we are able to determine all primes in a block of 150,000 numbers in a single run. The r_i's are found by direct division if $p_i \leqq (2p_n^{11/3} + 1)$ and by Alway's method [5] if otherwise. The time required for such a run is very little larger than that required to establish one prime in the same range.

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* Each available core location represents two odd numbers, differing by 75,000 and is initially set to a flagged record mark. If in the sieve, the lower number is found to be composite, the record mark is cleared, and if the higher number is so found, the flag is cleared.